# PRECISION MEASUREMENT OF THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

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The muon's magnetic moment anomaly, and hence $g$-factor anomaly $a = \frac{1}{2}(g-2)$, has been measured in the CERN muon storage ring. Final results are $a(\mu^-) - a(\mu^+) = (50 \pm 75) \times 10^{-8}$, and $a(\mu) = (116616 \pm 31) \times 10^{-8}$, which is 1.8 standard deviations above the theoretical value $a_{th}(\mu) = 116560 \times 10^{-8}$.

The anomalous part of the muon $g$-factor, $a \equiv \frac{1}{2}(g-2)$, is calculated from standard quantum electrodynamics [1].

$$a = \alpha/2\pi + 0.7658\ \alpha^2/\pi^2 + 2.49\ \alpha^3/\pi^3 + \ldots$$
$$= 11655.7 \times 10^{-7} \tag{1}$$

where the $\alpha^3$ -term is obtained from an estimate of the sixth-order contribution to the electron anomaly [2] ($0.13\ \alpha^3/\pi^3$), plus a calculation of all sixth-order terms which are different for the muon [3], and using [4] $\alpha^{-1} = 137.0359$.

Adding a contribution from strong interactions [5], the current theoretical prediction is

$$a_{\text{theory}} = 11656.0 \times 10^{-7} \tag{2}$$

with an uncertainty (due to strong interactions) of less than $1 \times 10^{-7}$. Possible weak interaction contributions have also been considered [6].

We determine the anomalous part of the magnetic moment with the following experimental arrangement. Injection of polarized muons into the weak-focusing ring magnet ($\rho = 250$ cm, $B = 17.11$ kG, $p_\mu = 1.27$ GeV/$c$) is accomplished by the forward decay of pions produced when a target in the magnetic field is struck by 10.5 GeV protons from the CERN Proton Synchrotron [7] (see fig. 1). The polarization direction of the muons as a function of time is followed by recording electrons emitted by muon decay in flight. These electrons emerge on the inside of the ring and produce a large pulse in lead-scintillator-sandwich shower detectors. The detection electronics is biased to accept only the highest energy electrons which must come from forward decay in the muon rest frame. As a result, the counting rate is modulated by the muon precession due to the anomalous moment at frequency

$$\omega_a = a(e/m_0c)\bar{B}\ , \tag{3}$$

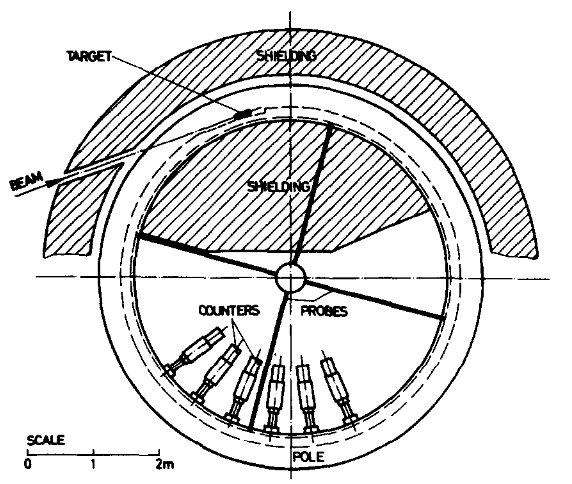


Fig. 1. Plan of 5-metre diameter ring magnet. The proton beam enters through a hole in the yoke and hits a target in the magnetic field. Muons created by $\pi$-$\mu$ decay are stored. The counters detect electrons from $\mu$-e decay. The four probes of the NMR magnetometers are injected into specific locations on the muon orbit every 200 cycles of the PS.

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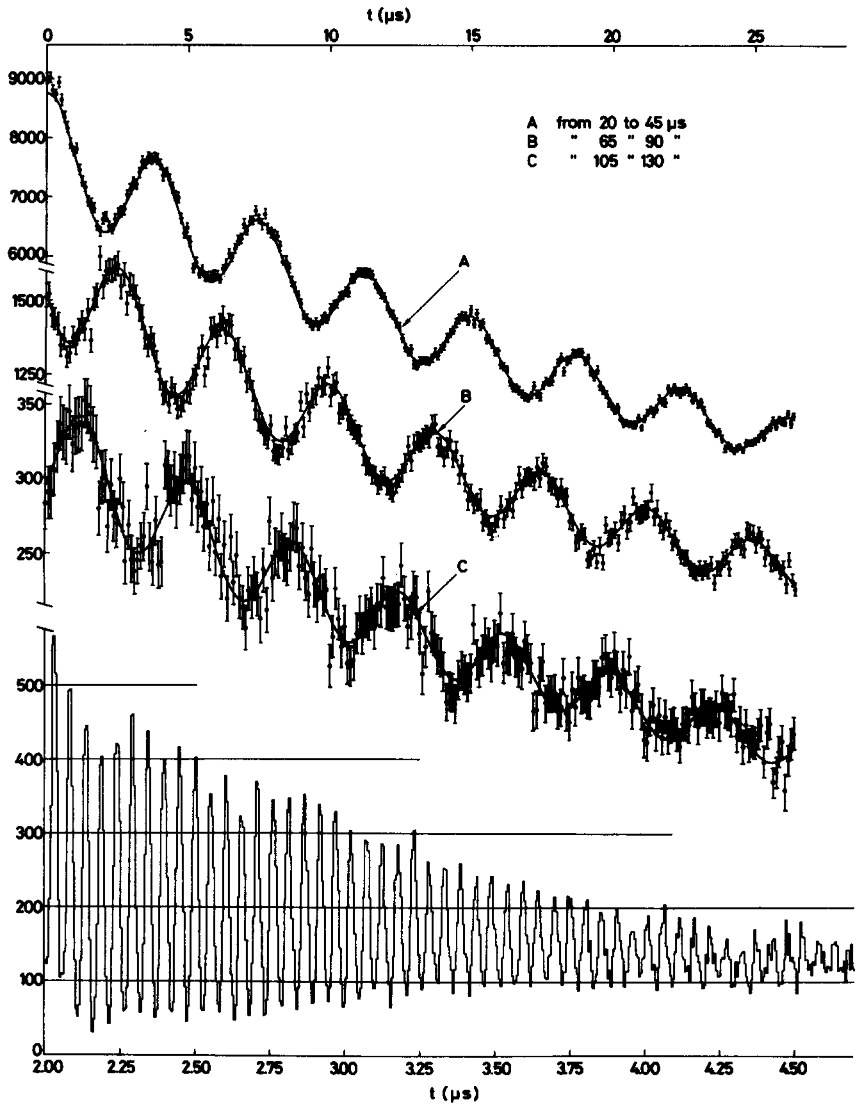


Fig. 2. Distribution of decay-electron events as a function of time. Lower curve shows rotation frequency of muon at early time. A, B, C: late time data, 20 - 130 $\mu$sec showing ($g$ - 2)-precession. Data are fitted from 21 to 190 $\mu$sec.

where $\bar{B}$ is the time average magnetic field for the ensemble of muons which contribute to the data [e.g. 8]. The magnetic field is measured in terms of the corresponding mean proton precession frequency $\bar{\omega}_p$.

The value of $(e/m_0c)$ for the muon is obtained from the ratio of muon to proton precession frequency in the same field, $\lambda \equiv \omega_\mu/\omega_p$. We have

$$(e/m_0c)\,\bar{B} = \lambda(1+\epsilon)(1+a)^{-1}\,\bar{\omega}_p\,. \quad (4)$$

$\lambda$ has been measured by Hutchinson et al. [9] for muons stopped in water relative to protons in water. In our case the muons are in vacuum, whilst the proton environment is as before. The correction $(1+\epsilon)$ is the diamagnetic shielding of muons in water, relative to vacuum, calculated by Ruderman [10].

Equations (3) and (4) yield, for the ratio $(\omega_a/\bar{\omega}_p)$ which we measure,

$$\omega_a/\bar{\omega}_p = a(1+a)^{-1}\,\lambda(1+\epsilon)\,. \qquad (5)$$

As the radial magnetic gradient necessary for vertical focusing implies a field variation of 0.4% over the 8 cm radial aperture of the muon storage region, a major problem is to determine the mean radius of the ensemble of muons which contribute to our counts. This has been obtained by measurements related to the rotation frequency of the muons. As the injection pulse is only 5 - 10 nsec long, and the rotation period $T = 2\pi\rho/\beta c$ is about 52.5 nsec, the muons are initially bunched, and the counting rate in the decay-electron counters is modulated during the first few microseconds (see lower part of fig. 2). Analysis of the modulated records yields the mean radius to $\pm 2$ mm, as shown by tests on Monte-Carlo-generated data.

Experimental checks indicate that the method is valid. However, a 25% loss of particles occurs between the measurement of $T$ and the measurement of $\omega_a$, raising the possibility of a change of radius. Experiments where we restricted the horizontal aperture from the inside and from the outside of the ring indicate that the mean radius does not change with time by more than 1 mm. We assign an overall error of $\pm 3$ mm in radius.

We fit the data with a function that allows for the time dependence of the losses. The results vary less than $\pm 0.2$ standard deviation as a function of the starting time of the fit.

The statistical error is $\pm 2.3 \times 10^{-7}$. The fluctuations of the results of eight different runs about the mean gives $\chi^2 = 7.84$ compared with 6.35 expected. The random error has been increased, by the square root of this ratio, to $\pm 2.5 \times 10^{-7}$.

To this must be added the error in magnetic field corresponding to $\pm 3$ mm uncertainty in radius, discussed above: this contributes $\pm 1.9 \times 10^{-7}$ error in $a$. These two errors are combined quadratically to give the overall error of $\pm 3.1 \times 10^{-7}$.

The result is

$$a_{\text{exp}} = (11661.6 \pm 3.1) \times 10^{-7}$$

or

$$a_{\text{exp}} - a_{\text{theory}} = +(5.6 \pm 3.1) \times 10^{-7}$$
$$= +(480 \pm 270)\ \text{ppm}^{*}\,.$$

This result tests QED to new levels of precision. For example, a conventional high momentum-transfer cut-off [11] of $\Lambda \sim 5$ GeV/$c$ would *lower* the theoretical result by 270 ppm and this appears unlikely. A coupling of the muon to an unknown boson field [12] could cause either an increase or decrease.

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* Runs with positive and negative muons, analysed separately, as a check on CPT-invariance, give the following results:

$$a_{\mu^+} = (11657.5 \pm 7.1) \times 10^{-7}$$
$$a_{\mu^-} = (11662.5 \pm 2.4) \times 10^{-7}$$

or

$$a_{\mu^-} - a_{\mu^+} = (5.0 \pm 7.5) \times 10^{-7}$$

where the errors are only statistical.

† In general only the latest publications are given; other references will be found in the papers sited.

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